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# Discretionary Policy in a Small Open Economy: Exchange Rate Regimes and Multiple Equilibria <sup>\*</sup>

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## Abstract

We study a Markov-perfect monetary policy in an open New Keynesian economy with incomplete financial markets. We analyze inflation and exchange rate targeting regimes and demonstrate that both cases may result in multiple equilibria. These equilibria feature qualitatively and quantitatively different economic dynamics following the same shock. The model can help us to understand sudden changes in the interest rate and exchange rate volatility in ‘tranquil’ and ‘volatile’ times under a fully credible ‘soft peg’ of the nominal exchange rate.

Key Words: Small open economy, Incomplete financial markets, Discretionary Monetary policy, Multiple Equilibria

JEL References: E31, E52, E58, E61, C61,F4

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# 1 Introduction

Choosing the exchange rate regime for an open economy is one of the classic macroeconomic problems. The conventional idea behind an exchange rate peg is that it will anchor inflation expectations and increase trade through lower uncertainty and smaller adjustment costs. It may also encourage investment into long-term projects due to lower exchange rate risk/ transaction costs and therefore has a positive economic impact (see e.g. Cote (1994) and Prasad, Rogoff, Wei, and Kose (2003) for the potential benefits of fixed exchange rate regimes). However, being prone to speculative attacks hard pegs became less popular, especially after the Asian crisis of 1997. On the other hand recent evidence suggests that monetary authorities in many developing countries still see targeting the nominal exchange as a priority, despite that they officially claim to have floating regimes.<sup>1</sup> Developing and emerging countries like Indonesia, Malaysia, Thailand, South Korea, Turkey, Russia adopted *de jure* flexible exchange rate regimes, but *de facto* the exchange rate remained one of the most important if not the only target of their monetary policy in the post 1997 decade.<sup>2</sup>

Despite a relatively tranquil post-1997 decade in most developing and emerging countries, the exchange rate volatility under these ‘soft pegs’ varied over time. There is also a number of studies that document difficulties in explaining sudden changes in ‘regimes’ between periods of high and low volatilities.<sup>3</sup> Theoretical explanations for these different regimes include non-rational behavior, non-linear decisions or heterogeneity of agents like the presence of ‘noise traders’ (see Jeanne and Rose (2002) for an important example).

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<sup>1</sup>See e.g. Levy-Yeyati and Sturzenegger (2005) and Calvo and Reinhart (2002).

<sup>2</sup>See Rahmatsyah, Rajaguru, and Siregar (2002) for Thailand, Dogolnar (2002) for Turkey, Korea, Malaysia, Indonesia and Pakistan, and Arize, Osang, and Slottje (2000) for 13 developing countries. Furthermore, evidence by Reinhart and Rogoff (2004) suggests that partial exchange rate targeting is still the predominant monetary policy regime in many developing countries.

<sup>3</sup>See e.g. Engel and Hamilton (1990), Clarida et al. (2003) or Chen (2006) who apply Markov-switching models to explain these changes. These models have also been employed to describe exchange rate behaviour in floating regimes. However, their success is still a matter of debate see e.g. again Clarida et al. (2003) and Engel, Mark, and West (2007).

In this paper we offer a different theoretical approach which can contribute to the explanation of these empirical observations. Using a simple small open economy model in the spirit of Galí and Monacelli (2005) but with incomplete financial markets (Benigno, 2009) we demonstrate that *the way* monetary policy is conducted might be responsible for the existence of time periods with large difference in the volatility of all macroeconomic variables, including the exchange rate.

Specifically, we demonstrate that discretionary monetary policy may result in multiple equilibria, consistent with different sets of beliefs of the private sector and the policymaker. A policy maker responds to a state that is at least partly determined by a forecast of his behavior. Different sets of beliefs about the future policy generate different future courses for a policy maker to follow. Therefore, if the economy is hit by a shock, it can follow one of several adjustment paths, where the volatility along these paths is different, resulting in different welfare outcomes.<sup>4</sup>

Once multiple equilibria are observed, coordination on the best equilibrium may be difficult. Discretionary policy is time-consistent by construction, so if the current policymaker perceives a particular – even inferior – policy of future policymakers, it will be optimal for the policymaker to implement the same policy in the current period. A unilateral deviation of the current policymaker from the perceived policy plan is not beneficial to this policymaker, and the resulting discretionary policy is a Nash equilibrium in the game of consequent policymakers, see e.g. Oudiz and Sachs, 1985; Dennis and Kirsanova, 2017. In other words, the existence of multiple equilibria implies that consequent policymakers may fail to coordinate on the best equilibrium or, equivalently, fall into an expectation trap.<sup>5</sup>

This paper compares and contrasts inflation targeting with soft exchange rate targeting under discretionary policy. Multiplicity arises in both cases. Under conventional inflation targeting the model has three stable discretionary equilibria. If the economy is hit by a cost-push shock, in two

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<sup>4</sup>The existence of multiple equilibria under discretionary policy in non-linear models has been well established by King and Wolman (2004) and Albanesi et al. (2003). Blake and Kirsanova (2012) show that multiplicity can also occur in LQ RE models under discretionary policy.

<sup>5</sup>See Cooper and John (1988) for a detailed discussion about coordination failures and multiple equilibria.

of the three equilibria the monetary policy maker raises the interest rate and reduces demand. Although the terms of trade improve, their positive effect on marginal costs is dominated by the effect of lower demand. The private sector reduces its holdings of net foreign assets creating a persistent deficit. The deficit and all macroeconomic variables converge back to the steady state at a very slow rate. In contrast, in the second equilibrium the cost push shock is initially accommodated, and the large depreciation of the terms of trade causes the net foreign assets to accumulate. Once the level of assets is high, the interest rate is raised and inflation is brought back to its base. This delayed increase in the interest rate also brings net foreign assets – and all macroeconomic variables – back to the base line very quickly. In the presence of multiple equilibria, a coordination failure occurs: the agents can choose any of them and a sunspot decides which one will realize.

A similar coordination failure may also happen under exchange rate targeting, either partial or strict. The policy maker introduces an additional positive weight – in an otherwise standard policy objective – that punishes the volatility of the nominal exchange rate. There is still a conventional discretionary equilibrium in which the exchange rate remains on target. However, as it is acceptable – but costly – that the future exchange rate can deviate from the target, the economic agents may coordinate on the second admissible equilibrium, in which the exchange rate is volatile around the target.

The assumption of incomplete financial markets is crucial for our results. A discretionary policy maker takes current and future economic conditions into account, but can only commit to current behavior. An endogenous predetermined state is a vehicle through which the current economic condition is affected by the past behavior of the rational private sector which is again based on a forecast of future economic conditions and future policy. Blake and Kirsanova (2012) demonstrate that the presence of such a state is a necessary condition for a multiplicity of equilibria to arise.

Our model abstracts from many features that may characterize developing countries, e.g. capital controls or incomplete exchange rate pass-through. Also, we use a relatively standard parameterization of model parameters rather than estimate them and, as such, present numerical examples, not quantitative statements with any empirical content. However, we show that the assumption of incomplete financial markets in combination with discretionary monetary policy is sufficient to generate ‘expectation traps’, i.e. multiple policy equilibria which are associated with different volatilities of all macroeconomic variables. Our results are not restricted to this simple model, but will also prevail in more detailed settings. Both assumptions – incomplete financial markets and discretionary monetary policy – are justifiable for developing countries: Such countries have restricted access to international financial markets and the ability of policymakers to precommit to future policies is generally weaker than in developed countries.

The remainder of the paper is organized as follows. The next Section presents the model. Section 3 discusses the policy equilibria for an inflation targeting regime and under nominal exchange rate targeting. Section 4 concludes.

## **2 Model Highlights**

We use a workhorse small open economy model, based on Galí and Monacelli (2005) and De Paoli (2009), with incomplete financial markets as in Benigno (2009). Specifically, there are two countries: the small open economy (Home) and the rest of the world (Foreign). The size of the Home economy is infinitely small relative to the size of the Foreign economy, therefore the economic performance and policy decisions do not have any impact on the rest of the world. Each economy is populated by infinity-living households and firms. Households’ consume two goods, home- and foreign-produced, and their preferences reflect home bias in consumption. The law of one price holds. Firms are monopolistically competitive, and only use labor to produce differentiated tradable goods. Production takes place in two stages. First, there is a continuum

of intermediate goods firms, which produce a differentiated input. In the second stage final goods producers combine these inputs into output and sell them to households in both countries. Monopolistic competition and sticky prices give a meaningful role for monetary policy. Each country has an independent fiscal authority, which finances spending by distortionary taxes and bonds. Home bonds are not tradable and in zero net supply, while foreign bonds are internationally tradable. Financial markets are incomplete, and the portfolio allocation is determined by transaction costs. All profits received by home country firms and financial intermediaries are rebated to home households. The Home country is subject to cost-push shocks. Full details of underlying microfoundations of the model are given in Appendix A. As we demonstrate in Appendix A, the model has unique deterministic steady state. In the following we present only the linearized equations around it.

## 2.1 Private Sector Equilibrium

The household optimization problem for the small open economy  $H$  yields a consumption Euler equation

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} + \sigma \alpha \left( \mathbb{E}_t \hat{S}_{t+1} - \hat{S}_t \right) - \sigma \left( \hat{i}_t - \mathbb{E}_t \pi_{Ht+1} \right), \quad (1)$$

where  $\hat{c}_t$  denotes consumption,  $\hat{S}_t$  is the terms of trade (relative price of foreign producer price in terms of home producer price),  $\hat{\pi}_{Ht}$  is Home producer price inflation and  $\hat{i}_t$  is the short term nominal interest rate.<sup>6</sup> Parameter  $\sigma$  is the inverse of the intertemporal elasticity of substitution and  $\alpha$  is the degree of trade openness.

The firms' optimization problem gives the standard New Keynesian Phillips curve for the producer price inflation

$$\hat{\pi}_{Ht} = \hat{v}_t + \frac{\lambda}{\left(1 + \frac{\tau^l}{\mu}\right)} \left( \alpha \hat{S}_t + \frac{1}{\sigma} \hat{c}_t + \left( \varsigma - \frac{\tau^l}{\mu} \right) \hat{y}_t + \frac{\tau^l}{\mu} \hat{g}_t \right) + \beta \mathbb{E}_t \hat{\pi}_{Ht+1}, \quad (2)$$

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<sup>6</sup>Here and below, hatted variables indicate that they have been linearized relative to their steady states. Steady state variables are denoted by letters without time subscript.

where  $\hat{y}_t$  is output,  $\hat{g}_t$  is government spending, financed by distortionary taxes, and  $\hat{v}_t$  is an AR(1) Home cost-push shock. Parameter  $\varsigma$  is the inverse of the Frisch elasticity of labor supply and  $\mu = \frac{\epsilon}{\epsilon-1}$  is a monopolistic markup which is related to the elasticity of substitution between home goods  $\epsilon$ . Parameter  $\beta$  denotes the household discount factor and the slope of the Phillips curve  $\lambda = (1 - \theta\beta)(1 - \theta)/\theta$  is a function of the Calvo (1983) probability of price change  $\theta$ . Finally, parameter  $\tau^l$  is the steady state value of the labour income tax. The Home country maintains a balanced budget, so the effect of marginal cost on inflation is scaled by the factor  $1/\left(1 + \frac{\tau^l}{\mu}\right)$ .

The aggregate resource constraint can be written as

$$\hat{y}_t = (1 - \alpha) \frac{c}{y} \hat{c}_t + \eta \alpha \left( (1 - \alpha) \frac{c}{y} + \frac{c^*}{y} \right) \hat{S}_t + \alpha \frac{c^*}{y} \hat{c}_t^* + \frac{g}{y} \hat{g}_t. \quad (3)$$

where  $\hat{c}_t^*$  is Foreign consumption. Parameter  $\eta$  is the elasticity of substitution between home and foreign goods.

For the other, large and effectively closed economy, the corresponding equations are

$$\hat{c}_t^* = \mathbb{E}_t \hat{c}_{t+1}^* - \sigma \left( \hat{i}_t^* - \mathbb{E}_t \pi_{Ft+1}^* \right), \quad (4)$$

$$\hat{y}_t^* = \frac{c^*}{y^*} \hat{c}_t^* + \frac{g^*}{y^*} \hat{g}_t^*, \quad (5)$$

$$\hat{\pi}_{Ft}^* = \lambda \left( \frac{1}{\sigma} \hat{c}_t^* + \varsigma \hat{y}_t^* + \frac{\tau^{*l}}{\mu} \hat{\tau}_t^{*l} \right) + \beta \mathbb{E}_t \pi_{Ft+1}^*, \quad (6)$$

$$\hat{b}_t^T = \varrho \hat{i}_t^* + \frac{1}{\beta} \left( \hat{b}_{t-1}^T - \varrho \hat{\pi}_{Ft}^* + \frac{g^*}{y^*} \hat{g}_t^* - \frac{\tau^{l*}}{\mu} \left( \hat{\tau}_t^{l*} + \frac{1}{\sigma} \hat{c}_t^* + (\varsigma + 1) \hat{y}_t^* + \frac{\tau^{*l}}{\mu} \hat{\tau}_t^{*l} \right) \right), \quad (7)$$

where  $\hat{b}_t^T$  is normalized total real Foreign debt issued by the Foreign government and held by Home and Foreign residents, parameter  $\varrho = \frac{b_F^T}{y^*}$ .

Finally, the model is closed with the risk premium equation

$$\hat{i}_t = \hat{i}_t^* + \mathbb{E}_t \pi_{Ht+1} - \mathbb{E}_t \hat{\pi}_{Ft+1}^* + \mathbb{E}_t \hat{S}_{t+1} - \hat{S}_t - \delta \beta \left( \hat{b}_t + \kappa (1 - \alpha) \hat{S}_t \right), \quad (8)$$



the current account equation

$$0 = \left( \alpha \frac{c}{y} \frac{y}{y^*} (1 - \eta(1 - \alpha)) - \frac{c^*}{y^*} \alpha \eta - (1 - \beta) \kappa \right) S_t + \alpha \frac{c}{y} \frac{y}{y^*} \hat{c}_t + \beta b_t - b_{t-1} - \alpha \frac{c^*}{y^*} \hat{c}_t^* - \kappa \beta \hat{i}_t^* + \kappa \hat{\pi}_{Ft}^* \quad (9)$$

and the definition of nominal exchange rate  $\hat{E}_t$

$$\hat{E}_t = \hat{S}_t - \hat{P}_{Ht} + \hat{P}_{Ft}^*, \quad (10)$$

where  $\delta$  is a Home portfolio adjustment cost parameter,  $\kappa = \frac{b_F}{y^*}$  is a measure of Foreign debt held by non-residents in the steady state.

The system of ten equations (1)-(10) describes the private sector equilibrium and determines  $\hat{c}_t, \hat{y}_t, \hat{\pi}_{Ht}, b_t, b_t^T, \hat{c}_t^*, \hat{y}_t^*, \hat{\pi}_{Ft}^*, \hat{E}_t$  and  $\hat{S}_t$ , given the policy variables  $\hat{i}_t, \hat{i}_t^*, \hat{g}_t, \hat{g}_t^*, \hat{\tau}_t^{l*}$ .

## 2.2 Small Open Economy Model in LQ Policy Framework

In the following we will only consider the dynamics of the small open economy and treat all variables of the large closed economy as exogenous shocks. Hence, we only work with the system of equations (1)-(3) and (8)-(10). Endogenous variables are  $\hat{c}_t, \hat{y}_t, \hat{\pi}_{Ht}, b_t, \hat{E}_t$  and  $\hat{S}_t$ , the policy instrument is  $\hat{i}_t$ . The cost push shock  $\hat{v}_t$  as well as government spending  $\hat{g}_t$  is treated as exogenous. System (4)-(7) determines other exogenous processes  $\hat{c}_t^*, \hat{y}_t^*, \hat{\pi}_{Ft}^*, b_t^T, \hat{i}_t^*, \hat{g}_t^*, \hat{\tau}_t^{l*}$ .

System (1)-(3), (8)-(10) can be represented in the following form, suitable for standard policy analysis in a linear-quadratic (LQ) framework. For convenience we introduce a new variable

$$u_t = \hat{c}_t - \sigma(1 - \alpha) \hat{S}_t \quad (11)$$

which measures the excess consumption under incomplete financial markets, as  $u_t \equiv 0$  under international risk sharing. Substituting out consumption and the interest rate (using equation (8)), the consumption Euler equation can be re-written as

$$u_t = \mathbb{E}_t u_{t+1} + \delta \sigma \beta \left( b_t + \kappa(1 - \alpha) \hat{S}_t \right) - \sigma \left( \hat{i}_t^* - \hat{\pi}_{Ft+1}^* \right). \quad (12)$$

The Phillips curve becomes

$$\begin{aligned}\hat{\pi}_{Ht} = & \hat{v}_t + \frac{\lambda \left( 1 + (\varsigma - \Theta_H) \left( (\sigma(1 - \alpha) + \eta\alpha)(1 - \alpha) \frac{c}{y} + \eta\alpha \frac{c^*}{y} \right) \right)}{\left( 1 + \frac{g}{y} \right)} \hat{S}_t \\ & + \frac{\lambda \left( \frac{1}{\sigma} + \left( \varsigma - \frac{g}{y} \right) (1 - \alpha) \frac{c}{y} \right)}{\left( 1 + \frac{g}{y} \right)} u_t \\ & + \frac{\lambda \left( \left( \varsigma - \frac{g}{y} \right) \alpha \frac{c^*}{y} \hat{c}_t^* + \frac{g}{y} \left( \varsigma + 1 - \frac{g}{y} \right) \hat{g}_t \right)}{\left( 1 + \frac{g}{y} \right)} + \beta \mathbb{E}_t \hat{\pi}_{Ht+1}\end{aligned}\quad (13)$$

and the current account equation is given by

$$\begin{aligned}b_t = & \frac{1}{\beta} b_{t-1} + \frac{1}{\beta} \left( (1 - \beta) \kappa + \alpha \left( \frac{c^*}{y^*} \eta - (1 - (\eta - \sigma)(1 - \alpha)) \frac{c}{y} \frac{y}{y^*} \right) \right) S_t \\ & - \frac{\alpha c}{\beta y y^*} u_t + \frac{1}{\beta} \left( \alpha \frac{c^*}{y^*} \hat{c}_t^* + \kappa (\beta \hat{i}_t^* - \hat{\pi}_{Ft}^*) \right).\end{aligned}\quad (14)$$

In the following we will treat the terms of trade  $\hat{S}_t$  as policy instrument, as it only enters contemporaneously in equations (12)-(14).<sup>7</sup> The three endogenous variables in system (12)-(14) are foreign debt  $b_t$ , producer-price inflation  $\hat{\pi}_{Ht}$  and excess consumption  $u_t$ . Once this system is solved, the interest rate needed to deliver the optimal policy can be found from (8), consumption can be recovered from equation (11) and output can be found from the aggregate demand equation (3).

## 2.3 Monetary Policy Regimes

### 2.3.1 Inflation Targeting

In this paper we assume that the following quadratic policy objective is delegated to the central bank by either society or legislation (see e.g. Kam et al. (2009)):

$$W_t^{IT} = \frac{1}{2} \mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} (\pi_{H,s}^2 + \omega_y \hat{y}_s^2), \quad (15)$$

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<sup>7</sup>This approach is common in the literature, see Clarida et al. (1999) where consumption is treated as policy instrument and the interest rate is later reinstated from the Euler equation. Our approach is similar. Once the solution is found, the interest rate can be reinstated from the risk premium equation.

where  $\beta_m$  is monetary policymaker's discount factor. The above policy objective has been shown by Woodford (2003) to approximate the aggregate of individual utility functions in a closed economy model with complete financial markets. In our model, this approximation will not hold up to the second order, but is frequently considered in the literature as a likely objective given to the central bank. Where relevant, this policy objective also plays the role of 'social loss', so all welfare losses are computed using metric (15).<sup>8</sup>

### 2.3.2 Nominal Exchange Rate Targeting

Under nominal exchange rate targeting the central bank uses the following objective

$$W_t^E = \frac{1}{2} \mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} \left( (1 - \omega_e) (\pi_{H,s}^2 + \omega_y \hat{y}_s^2) + \omega_e \hat{E}_s^2 \right), \quad (16)$$

where we impose an additional weight  $\omega_e$  on the stabilization of the nominal exchange rate around its steady state value.

If  $\omega_e = 0$  then the objective (16) reduces to the standard inflation targeting regime (15). If  $\omega_e = 1$  then the objective (16) is equivalent to a strict exchange rate targeting regime:

$$W_t^{E1} = \frac{1}{2} \mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} \hat{E}_s^2. \quad (17)$$

This targeting regime has some similarities with a fixed exchange rate regime. In particular, this regime assumes that the policy maker announces the target, perhaps within a corridor (which we do not model as binding in any way, so it does not affect expectations of the private sector) and implements policy to keep the exchange rate on target. The exchange rate, however, is allowed to deviate from the target, although such deviations are costly.<sup>9</sup>

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<sup>8</sup>Appendix B provides the robustness analysis to this form of policy objective.

<sup>9</sup>The peg is 'soft' as it is a result of optimisation policy and so is different from a 'hard' peg where the monetary policy maker is prepared to sell any quantity of reserves at a given price to keep the exchange rate *exactly* on target. A hard peg cannot be modeled within our framework of optimisation with a quadratic loss function because any regime with quadratic loss function allows (costly) deviations from the parity.

### 2.3.3 Policy, Policy Instrument and Solution

The central bank manipulates the short term interest rate to affect the terms of trade  $\hat{S}$  to minimize loss (15) (or (16)) subject to system(12)-(14). A discretionary solution can be written in the form of linear feedback rules

$$b_t = b_y \bar{y}_t + b_x \bar{x}_t, \quad (18)$$

$$u_t = U_y \bar{y}_t + U_x \bar{x}_t, \quad (19)$$

$$\hat{\pi}_{H,t} = \Pi_y \bar{y}_t + \Pi_x \bar{x}_t, \quad (20)$$

$$\hat{S}_t = s_y \bar{y}_t + s_x \bar{x}_t \quad (21)$$

where  $\bar{y}_t$  denotes a vector of endogenous predetermined states, and  $\bar{x}_t$  denotes a vector of exogenous predetermined states. In case of inflation targeting vector  $\bar{y}_t = [b_{t-1}]$  is a scalar, while in case of nominal exchange rate targeting it is  $\bar{y}_t = [p_{Ht-1}, b_{t-1}]$ . Vector  $\bar{x}_t$  contains the stochastic component of the solution  $\bar{x}_t^{IT} = [\hat{v}_t]$  for all policy regimes we consider.<sup>10</sup> A representation in the form of (18)-(21) is convenient to illustrate the multiplicity of discretionary equilibria.

### 2.4 Parameterization

The share of government spending to GDP,  $g/y$  and  $g^*/y^*$ , is set to 0.20 for each country. We set  $\omega = \frac{b_F}{b_F^T} = 0.0$  as the small open economy is unlikely to be a substantial non-resident holder of the large country's debt. We set  $\varrho = \frac{b_F^T}{y^*} = 0.6 \times 4$  which reflects a 60% annual debt to output ratio. Parameters  $g/y$ ,  $g^*/y^*$ ,  $\varrho$  and  $\omega$  yield the steady state tax level needed to service debt  $\frac{\tau^l}{\mu} = \frac{g}{y}$  and  $\frac{\tau^{*l}}{\mu} = \frac{g^*}{y^*} + \varrho(1 - \beta)$ . Finally, we set the adjustment cost parameter  $\delta = 0.01$  following Benigno (2009). This implies a 10 basis point spread of the domestic interest rate over the foreign one. We assume that the policymaker's discount factor  $\beta_m = \beta$ .

The model frequency is quarterly. The household's discount factor  $\beta$  is set to 0.99 which gives

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<sup>10</sup> Coefficients  $\bar{b}, \bar{u}, \bar{\pi}, \bar{s}$  can be found by solving the first order conditions. Our definition of discretion is conventional, see e.g. Clarida et al. (1999).

the steady state interest rate of 4% and the Calvo parameter  $\theta$  is set to 0.75 which implies the average length of fixed price contracts of about one year. Openness is set to  $\alpha = 0.3$ . The inverse of the intertemporal elasticity is set to  $\sigma = 0.5$ , based on evidence in Attanasio and Weber (1995). The elasticity between home goods  $\epsilon = 11$  and the inverse of the Frisch elasticity of labour supply  $\zeta = 3$  are set consistently with most estimations of DSGE models (Liu and Muntaz (2011), Justiniano and Preston (2010), Chen et al. (2017)). The intertemporal elasticity of substitution between domestic and foreign goods  $\eta$  is set to 1.5, see Albonico et al. (2016) and Adolfson et al. (2008). Finally, following Galí and Monacelli (2005) cost push shocks in the domestic country and the rest of the world follow AR(1) processes with persistence parameter  $\rho_v = 0.4$ . The standard deviation of a cost push shock is 0.005. We only consider one shock in our welfare computations, as this does not affect any of our results. Adding more shocks simply rescales the loss numbers in a not-informative way.

### 3 Discretionary Policy and Expectations Traps

In this section we demonstrate that expectation traps are relevant for a monetary policymaker in a small open economy.<sup>11</sup> We first consider standard inflation targeting policy. To stabilize inflation, the policymaker must choose a plan on how quickly to bring the marginal costs back to their steady state level. The policymaker may be expected to stabilize marginal costs slower or faster, and multiple equilibria arise. Similarly, under soft exchange rate targeting, different speeds of stabilization of the nominal exchange rate is possible.

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<sup>11</sup>This section presents numerical examples, not quantitative statements with any empirical content.

### 3.1 Inflation Targeting

#### 3.1.1 Discretionary Policy Equilibrium

Consider the deterministic version of the model. The dynamic system of equations (12)-(14) can be written as

$$\begin{aligned} u_t &= \mathbb{E}_t u_{t+1} + u_b b_t + u_s \hat{S}_t \\ \hat{\pi}_{Ht} &= \pi_s \hat{S}_t + \pi_u u_t + \beta \mathbb{E}_t \hat{\pi}_{Ht+1} \\ b_t &= \frac{1}{\beta} b_{t-1} + b_s S_t - b_u u_t \end{aligned}$$

where all coefficients are given in Appendix D. Under inflation targeting with policy objective (15) optimal time-consistent control variables can be written in the form of (18)-(21):

$$\begin{aligned} u_t &= U_b b_{t-1}, \\ \hat{\pi}_{H,t} &= \Pi_b b_{t-1}, \\ \hat{S}_t &= s_b b_{t-1}. \end{aligned}$$

where coefficients satisfy the following Riccati equations

$$U_b = \tilde{U}_b + \tilde{U}_s s_b, \tag{22}$$

$$\Pi_b = \tilde{\Pi}_b + \tilde{\Pi}_s s_b, \tag{23}$$

$$V = \left( \tilde{\Pi}_b + \tilde{\Pi}_s s_b \right)^2 + \omega_s s_b^2 + \beta V \left( \frac{1}{\beta} - b_u \tilde{U}_b + \left( b_s - b_u \tilde{U}_s \right) s_b \right)^2, \tag{24}$$

$$s_b = - \frac{\left( \tilde{\Pi}_s \tilde{\Pi}_b + \beta V \left( b_s - b_u \tilde{U}_s \right) \left( \frac{1}{\beta} - b_u \tilde{U}_b \right) \right)}{\left( \beta V \left( b_s - b_u \tilde{U}_s \right)^2 + \tilde{\Pi}_s^2 + \omega_s \right)}, \tag{25}$$

$V$  denotes the value function matrix, and we introduced new notation using equations (22)-(23):

$$\begin{aligned}\tilde{\Pi}_b(U_b, \Pi_b) &: = \frac{\pi_u U_b + \beta (b_u u_b - b_u u_b \beta + 1) \Pi_b + \pi_u u_b \beta}{\beta (U_b b_u + b_u u_b + 1)}, \\ \tilde{\Pi}_s(U_b, \Pi_b) &: = \frac{(\pi_s + \pi_u u_s + (\pi_s b_u + \pi_u b_s) (u_b + U_b) + \beta (b_s - b_u u_s) \Pi_b)}{(U_b b_u + b_u u_b + 1)}, \\ \tilde{U}_b(U_b, \Pi_b) &: = \frac{U_b + u_b \beta}{\beta (U_b b_u + b_u u_b + 1)}, \\ \tilde{U}_s(U_b, \Pi_b) &: = \frac{u_s + U_b b_s + b_s u_b}{U_b b_u + b_u u_b + 1}.\end{aligned}$$

System (22)-(25) has four non-linear equations for four unknown coefficients,  $U_b, \Pi_b, V, s_b$ , which determine the dynamic solution to system (18)-(21). As the original optimization problem is certainty-equivalent, it is sufficient to solve the deterministic problem to find  $U_b, \Pi_b, V, s_b$  and then we can reinstate in the unique way all other coefficients in (18)-(21).<sup>12</sup>

Each solution  $\{U_b, \Pi_b, V, s_b\}$  to system (22)-(25) corresponds to a discretionary equilibrium.

### 3.1.2 Multiplicity of Policy Equilibria

Table 1 reports that system (22)-(25) has three distinct solutions, so that the model exhibits three discretionary equilibria under inflation targeting, which we label  $A$ ,  $B$ , and  $C$ . In the following we report the deterministic components of the solution and also reinstate the interest rate reaction function in the form  $\hat{i}_t = i_b b_{t-1}$ . The loss is computed for the stochastic model.

Table 1 shows that equilibria  $A$  and  $B$  share certain characteristics while equilibrium  $C$  appears very different.<sup>13</sup> In particular, the feedback coefficients on inflation, terms of trade and the nominal interest rate in equilibrium  $C$  are all much larger in magnitude than those for equilibria  $A$  and  $B$ , suggesting greater volatility in a stochastic economy. In addition, while the nominal interest rate is lowered in response to higher foreign assets in equilibria  $A$  and  $B$  it is raised markedly in equilibrium  $C$ . These three equilibria produce qualitatively and quantitatively different economic dynamics, as shown in Figure 1 which plots the responses of key variables to a

<sup>12</sup>See e.g. Blake and Kirsanova (2012).

<sup>13</sup>The loss is measured in percentage of steady state consumption which needs to be sacrificed to eliminate stochastic volatility.

Table 1: Multiple Discretionary Equilibria under Inflation Targeting

Eqm.	Policy Reaction	Private sector Reaction	Implied response of interest rate	Speed of adjustment	Value function	Average Loss
	$[s_b]$	$\begin{bmatrix} U_b \\ \Pi_b \end{bmatrix}$	$[i_b]$	$\lambda_{\max}$	$[V]$	$L, \%C$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A	-0.1106	$\begin{bmatrix} 0.1013 \\ 0.0008 \end{bmatrix}$	-0.0036	0.9534	8.9e-05	0.0166
B	-0.1258	$\begin{bmatrix} 0.0950 \\ -0.0867 \end{bmatrix}$	-0.0856	0.9505	0.0713	0.0715
C	-3.1888	$\begin{bmatrix} 0.0004 \\ -0.8100 \end{bmatrix}$	2.8586	0.0824	0.7127	0.4498

one-percent domestic mark up shock. To understand these results, and also to provide an intuition for the rise of multiplicity, we look closer at the transmission mechanism of shocks under optimal discretionary policy.

Monetary policy aims to stabilize inflation and does it via influencing the path for marginal cost. The forward representation of the Phillips curve (13) can be written as

$$\hat{\pi}_{H,t} = \lambda \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \hat{m}c_s + \frac{1}{1 - \beta \rho_v} \hat{v}_t, \quad (26)$$

where the real marginal costs can be expressed as<sup>14</sup>

$$\hat{m}c_s = w_1 \hat{S}_t + w_2 (b_{t-1} - \beta b_t) + w_3 \varepsilon_{d,t}. \quad (27)$$

It is apparent that movements in  $mc_t$  and  $mc_{t+1}$  are highly substitutable in terms of their effect on  $\pi_{Ht}$  and that there are multiple paths for  $mc_t$  that will return inflation to target. These different paths for real marginal costs are associated with different monetary policies and with different performances in terms of the loss. Equation (27) shows that monetary policy can affect

<sup>14</sup>Parameters  $w_1 > 0$ ,  $w_2 > 0$ ,  $w_3$  and the composite shock  $\varepsilon_{d,t}$  are given in Appendix C.

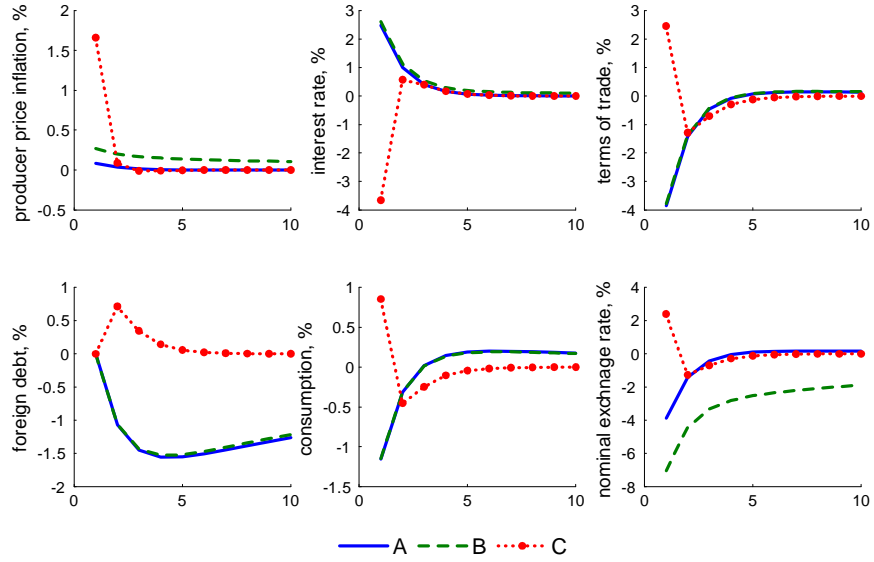


$mc_t$  through two distinct channels. The standard response to lower marginal cost is to tighten monetary policy. In our case this means an improvement in the terms of trade, as  $w_1 > 0$  in equation (27). Households will respond by reducing consumption and selling foreign assets as the terms of trade drive the evolution of net foreign assets in equation (14). Alternatively, the policy maker can conduct expansionary monetary policy which implies a depreciation of the terms of trade, but also causes net foreign assets to accumulate.

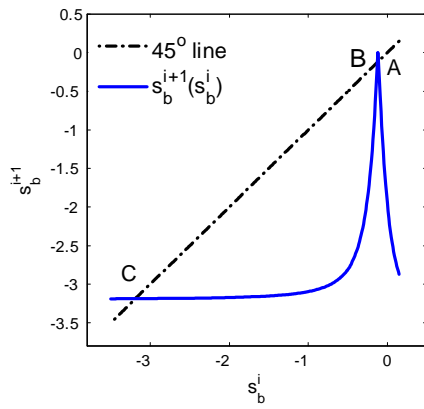
The key for multiplicity is that the impact of  $\hat{S}_t$  and  $b_t$  on  $\hat{m}c_t$  is in opposite directions. Notice that a reduction in  $\hat{S}_t$  causes a fall in  $b_t$  and that  $\hat{S}_t$  and  $b_t$  have countervailing effects on  $\hat{m}c_t$ . As a consequence, the desirability of each policy from the perspective of the period- $t$  policymaker depends on how future policymakers are expected to respond to movements in the stock of net foreign assets.

Consider the case where *future* policymakers are expected to lower the interest rate and depreciate the terms of trade in response to a rise in the stock of net foreign assets. Following a positive cost push shock  $\hat{v}_t$ , the *current* policy of raising the real interest rate and causing  $\hat{S}_t$  and  $b_t$  to decline will successfully deliver lower real marginal costs and inflation because the boost in future real marginal costs caused by the decline in the stock of net foreign assets is offset by lower terms of trade in the future. Under this approach, monetary policy responds to the positive markup shock by contracting demand, lowering real marginal costs and inflation. In subsequent periods lowering interest rates and increasing the terms of trade as inflation declines allows the economy to recover, producing an equilibrium. Alternatively, if *future* policymakers are expected to raise the interest rate and reduce the terms of trade in response to a higher stock of net foreign assets, then a *current* policy that lowers the real interest rate and raises the terms of trade can bring about a decline in inflation by stimulating the accumulation of foreign assets. This is despite the boost to  $\hat{S}_t$  and  $mc_t$  today, because future policymakers respond to the higher foreign assets by tightening monetary policy, producing another equilibrium.

Panel I: Responses to 1% cost push shock



Panel II: Multiple Equilibria



Panel III: Stabilization of NFA

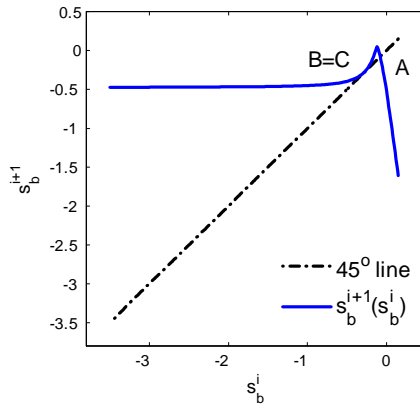


Figure 1: Inflation Targeting

This is illustrated in Panel I of Figure 1. As anticipated, we find that the behavior of the economy is notably different in equilibria  $A$  (and  $B$ ) and  $C$ . In equilibrium  $C$  monetary policy accommodates cost-push shocks, but allows to bring the stock of assets back to the base line quickly. The tight monetary policy in equilibrium  $A$  (and  $B$ ) results in a large reduction in the net foreign assets position with small but long-lasting effects on all macroeconomic variables.

These three equilibria are obtained numerically by searching for a fix-point in the policy-maker's response to the stock of foreign assets and plotted in Panel II of Figure 1. Given a perceived policy reaction function (21) with feedback on debt  $s_b^i$  we can compute the optimal reaction of the private sector  $(U_b^i, \Pi_b^i)$  using (22)-(23). Given the reaction of the private sector  $(U_b^i, \Pi_b^i)$ , we can solve the maximization problem of the central bank (24)-(24) to find the value function  $V$  and the best policy response,  $s_b^{i+1}$ . In points where  $s_b^{i+1} = s_b^i$  we have a discretionary equilibrium. There are three such points in Panel II of Figure 1.<sup>15</sup> All three equilibria are private-sector learnable, but only equilibria  $A$  and  $C$  are jointly learnable, as discussed in Dennis and Kirsanova (2017). This implies that conventional methods of finding discretionary solution by backwards induction (Oudiz and Sachs, 1985) can only find equilibria  $A$  and  $C$ .

### 3.1.3 Robustness and the Role of Financial Markets

The discovered equilibria are robust to different calibrations of the model. Changes in  $\sigma$ ,  $\varphi$  and  $b_H/y^*$  result in minor shifts of points of intersection in Panel II Figure 1, preserving qualitative differences in the dynamics of the different equilibria. The intermediation cost parameter  $\delta$  does not affect the equilibria in a significant way either. With  $\delta$  tending to zero transaction costs fall and the degree of financial integration rises. All three equilibria survive for any  $\delta > 0$ .<sup>16</sup> However, there is a discontinuity at  $\delta = 0$ , where the problem is isomorphic to the one under international risk sharing, with private sector investing into state-contingent assets and one-period foreign

<sup>15</sup>Appendix D gives a sketch of a formal proof that there are only three equilibria. It also presents all formal derivations and discusses the iterative procedure in detail.

<sup>16</sup>Checked numerically up to values  $\delta = 1e-8$ . See Appendix E.

bonds is one of them. A unique discretionary equilibrium with properties similar to those in equilibrium  $C$  can be obtained analytically. In this case equation (12) has unique solution  $u_t = 0$  and the greatly simplified system (22)-(25) has the unique solution<sup>17</sup>

$$\begin{aligned} U_b &= 0, \\ \Pi_b &= -\frac{\pi_s}{\beta b_s}, \\ V &= (\Pi_b + (\pi_s + \beta b_s \Pi_b) s_b)^2 + \omega_s s_b^2 + \beta V \left( \frac{1}{\beta} + b_s s_b \right)^2, \\ s_b &= -\frac{(\pi_s + \beta b_s \Pi_b) \Pi_b + V b_s}{\beta V b_s^2 + (\pi_s + \beta b_s \Pi_b)^2 + \omega_s}. \end{aligned}$$

With  $\delta = 0$  the dynamic system effectively de-couples into the Phillips curve and the risk sharing condition on the one hand, and the evolution of net foreign assets on the other hand. The evolution of net foreign assets becomes irrelevant for decisions of the private sector and the policymaker. Blake and Kirsanova (2012) demonstrate that for discretionary policy to generate multiplicity of equilibria it is necessary to have at least one endogenous predetermined state in the model. With  $\delta = 0$  the state becomes irrelevant and multiplicity disappears: equilibrium  $C$  remains, while equilibria  $A$  and  $B$  correspond to a single solution with zero feedback of all control variables on net foreign assets. Solutions  $A$  and  $B$  produce explosive dynamics of the economy, and are ruled out as discretionary equilibria by transversality conditions.<sup>18</sup>

### 3.1.4 Policy Delegation as Equilibrium Selection Mechanism

Can the multiplicity be eliminated by delegating a particular policy objective to the policymaker? The answer is positive, and many such delegation schemes may exist. However, they do not necessarily result in higher overall welfare. For example, the intrinsic property of equilibrium  $C$  is that it implies a relatively fast adjustment of the endogenous predetermined state, foreign assets. Making the fast adjustment costly may help to eliminate this equilibrium. Panel III in

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<sup>17</sup>More details are available in Appendix F.

<sup>18</sup>In this model we consider government debt. The results are unaffected by the debt ownership structure. Assuming that the foreign debt is privately-issued and its net supply is zero, would only affect exogenous processes.

Figure 1 illustrates a policy delegation scheme where the policymaker minimizes the loss with an additional penalty on changes in net foreign asset position

$$W_t^{IT,b} = \frac{1}{2} \mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} \left( \pi_{H,s}^2 + \omega_y \hat{y}_s^2 + \omega_b (b_t - b_{t-1})^2 \right). \quad (28)$$

Panel III demonstrates that with  $\omega_b > 0$  the line  $s_b^{i+1}(s_b^i)$  shifts up, so points  $C$  and  $B$  eventually disappear, once  $\omega_b$  is sufficiently large. However, this policy results in substantial welfare losses in the now equilibrium  $A$ . For our base line calibration,  $\omega_b \simeq 32$  results in an average loss of 0.4549, which is slightly greater than loss of now eliminated equilibrium  $C$  and substantially greater than the loss in equilibrium  $A$  with  $\omega_b = 0$ , see column (7) in Table 1.

Table 2: Effect of Discounting under Inflation Targeting

$\beta_m$	0.99	0.8	0.6	0.4	0.2	0.1	0.0
$A$	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166
$C$	0.4498	0.4499	0.4502	0.4512	0.4566	0.4751	–

In contrast to the policy delegation scheme discussed in (28), the loss in the best equilibrium is unaffected if the monetary policymaker is impatient,  $\beta_m < \beta$ , see Table 2 which reports the losses in equilibria  $A$  and  $C$  for different discount factors  $\beta_m$ .<sup>19</sup> Once the policymaker discounts the future at a sufficiently high rate ( $\beta_m \approx 0.08$  for the base line calibration), only the best equilibrium  $A$  survives.<sup>20</sup> Equilibrium  $C$  relies on the ability of the policymaker to delay stabilization of inflation until future periods, which is ruled out by impatience. Table 2 demonstrates, however, that the loss in the worst equilibrium  $C$  increases with the degree of impatience, and an inability to choose the right discounting may result in a substantially worse outcome.

<sup>19</sup>This result is conditional on the form of the objective function. The above objective function contains only forward-looking terms. Had we for example kept the change in NFA as a target, the loss in both equilibria would change with discounting. However, for illustrative purposes we report the result for the standard inflation targeting regime. Here and below we use the ‘social’ objective with discount factor  $\beta$  to calculate the loss.

<sup>20</sup>Numerical simulations produce a picture which is qualitatively similar to Panel 3 in Figure III.

### 3.2 Exchange Rate Targeting

Unlike inflation targeting, discretionary policy *can be consistent* with keeping the nominal exchange rate on target at all times.<sup>21</sup> If the policy maker targets the nominal exchange rate, complete stabilization can be achieved. However, this equilibrium is not unique and the policy-maker may not be able to achieve complete stabilization of the nominal exchange rate.

The intuition for this result is similar to the one presented above. Exchange rate targeting requires to stabilize prices of foreign goods. We can rewrite

$$\begin{aligned}\hat{E}_t &= P_{F,t} - P_t^* = \hat{S}_t + P_{H,t} - P_t^* = \hat{S}_t + \pi_{H,t} + P_{H,t-1} - P_t^* \\ &= \hat{S}_t + \lambda \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \hat{m}c_s + \frac{1}{1 - \beta \rho_v} v_t + P_{H,t-1} - P_t^*,\end{aligned}$$

which is similar to representation (26). Therefore, there is no surprise that we have several paths for marginal costs each of which eventually stabilizes the nominal exchange rate. Figure 2 demonstrates the responses to a domestic cost push shock in two discretionary equilibria labelled *A* and *C*.<sup>22</sup> Responses are qualitatively similar to the corresponding equilibria under inflation targeting plotted in Figure 1, Panel I. However, there are quantitative differences. In equilibrium *A* the policymaker reduces the terms of trade as little as needed to generate a small negative effect of marginal costs on inflation to match the increase in home price with the reduction in the terms of trade to keep the nominal exchange rate exactly on target. In this equilibrium inflation rises by much more than under inflation targeting in Figure 1, and this results in relatively large welfare losses, see Table 3.

In equilibrium *C* the policymaker lowers the interest rate sharply. The nominal exchange rate depreciates and is not kept on target. Nevertheless, this is a discretionary equilibrium consistent

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<sup>21</sup>Much of NOEM literature (see e.g. Gali and Monacelli (2005)) demonstrates that it is possible under interest rate rules.

<sup>22</sup>To obtain these equilibria we used the standard way of backwards induction (Oudiz and Sachs 1985). It allows us to obtain equilibria which are jointly learnable, see Dennis and Kirsanova (2017). Iterations on the policy reaction are not applicable, as we have more than one endogenous state variable. However, the discovered two equilibria are sufficient to illustrate multiplicity of equilibria.

with the stabilization of the nominal exchange rate, as the policymaker is still able to keep the nominal exchange rate stable ‘on average’ in response to a positive cost push shock. As soon as the private sector expects that future policymakers will raise the interest rate and reduce the terms of trade in response to a higher stock of net foreign assets, the current policymaker lowers the interest rate, raises the terms of trade and therefore stimulates an accumulation of foreign assets. It, therefore, delivers the expected future policy response, and validates the expectations of the private sector. The stabilization is costly, as the volatility of all variables is substantial. However, this strategy is consistent with the soft exchange rate target as it ensures the convergence back towards the target in the medium term, see Figure 2. In other words, as it is less costly to validate the expectations of the private sector than to accommodate them, the policymaker is trapped in this equilibrium. Similar to the inflation targeting regime the targeted variable is allowed to deviate from the target, and the current policymaker perceives that the private sector expects future policymakers to appreciate the nominal exchange rate, it will optimally choose to generate a depreciation of the nominal exchange rate today.

Table 3: Multiple Discretionary Equilibria under Strict Exchange Rate Targeting

Eqm.	Policy Reaction	Private sector Reaction	Implied response of interest rate	Speed of adjustment	Average Loss
	$\begin{bmatrix} s_p & s_d \end{bmatrix}$	$\begin{bmatrix} U_p & U_d \\ \Pi_p & \Pi_d \end{bmatrix}$	$\begin{bmatrix} i_p & i_d \end{bmatrix}$	$\lambda_{\max}$	$L, \%C$
(1)	(2)	(3)	(4)	(5)	(6)
$A$	$\begin{bmatrix} -0.6038 & -0.0409 \end{bmatrix}$	$\begin{bmatrix} -0.0462 & 0.1062 \\ -0.3962 & 0.0409 \end{bmatrix}$	$\begin{bmatrix} 0.0027 & -0.0097 \end{bmatrix}$	0.9532	0.1209
$C$	$\begin{bmatrix} 0.6537 & -2.7121 \end{bmatrix}$	$\begin{bmatrix} 0.0064 & -0.0053 \\ 0.0118 & -0.8257 \end{bmatrix}$	$\begin{bmatrix} -0.4247 & 1.0971 \end{bmatrix}$	0.6231	0.5437

Multiplicity is also preserved in the more general form of exchange rate targeting using the

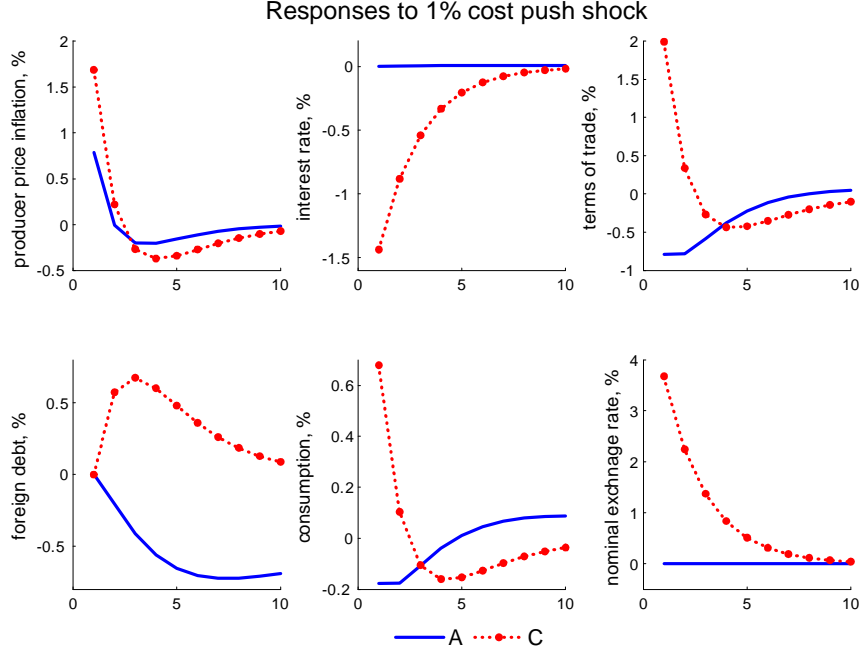


Figure 2: Soft Exchange Rate Targeting

hybrid objective (16). Table 4 reports losses for intermediate values of the relative weight on the exchange rate target  $\omega_e \in (0, 1]$ . If  $\omega_e = 0$  the objective (16) coincides with inflation targeting objective (15). The losses in equilibria with  $\omega_e > 0$  are higher than under inflation targeting.

Multiplicity, however, is eliminated if the policymaker is impatient. Equilibrium *C* arises as there is a possibility to stabilize the exchange rate tomorrow. Table 5 reports our results of reducing the policymaker's discount factor,  $\beta_m$  in the objective (17). Once it is sufficiently small ( $\beta_m \lesssim 0.03$ ) the stable inferior equilibrium disappears.<sup>23</sup> The best equilibrium is invariant to  $\beta_m$ .

Despite it is commonly suggested that currency pegging is an efficient way to import low and stable inflation, it is apparent that in the case of a 'soft peg' the implied volatility of the nominal exchange rate and domestic inflation in the worst regime is higher than it is in the case of inflation targeting. This is not surprising: these are two 'second-best' scenarios, and there cannot be any

<sup>23</sup>This result was obtained numerically. Appendix G demonstrates that for  $\beta_m = 0$  the nominal exchange rate is always kept on target.



Table 4: Multiplicity under Exchange Rate Targeting

$\omega_e$	1.0	0.8	0.6	0.4	0.2	0.05	0.02	0.0
$A$	0.1209	0.1195	0.1171	0.1126	0.1006	0.0588	0.0328	0.0166
$C$	0.5437	0.5429	0.5417	0.5394	0.5336	0.5161	0.5087	0.4498

Table 5: Effect of Discounting under Exchange Rate Targeting

$\beta_m$	0.99	0.8	0.6	0.4	0.2	0.05	0.03	0.0
$A$	0.1209	0.1209	0.1209	0.1209	0.1209	0.1209	0.1209	0.1209
$C$	0.5437	0.5478	0.6038	0.8907	1.2740	1.6124	1.6608	–

*a priori* ranking between them.

## 4 Conclusion

This paper demonstrates how multiple equilibria can occur in a small open economy model with incomplete financial markets under discretionary monetary policy. As current policymakers cannot control the behavior of future policymakers nor the expectation of the private sector coordination failures and expectation traps can occur. In our model policy makers need to decide if the economy should be stabilized today or at some point in the future.

We believe that the presented model is capable of explaining recent empirical evidence on exchange rate behavior: there can be switches between policy regimes that are characterized by changes in the volatility of the nominal exchange rate. This can happen for a wide and realistic class of policy objectives, as long as the policy maker acts under discretion and there is at least one predetermined state variable in the system.

Although the presented model is highly stylized, and bringing it to the data is therefore beyond the scope of this paper, a sufficiently complex model with these features will retain multiplicity of

equilibria and should be able to replicate the observed volatilities of key macroeconomic variables, in particular the nominal exchange rate.

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